The Quality & Cosmology of Nelson-Barr Models

[arXiv:2212.03882]

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CP Violation in the Standard Model

We all learn that CP violation arises from the presence of a physical *phase* in the Lagrangian.

For example, δ_{CKM} leads to CP violation in e.g., the Kaon system $\Longrightarrow$ *crucial* in building the structure of the SM, and now a powerful constraint on new physics.

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But there is another place where CP violation enters the SM — the theta term:

$$\frac{\theta}{16\pi^2}\int \mathrm{d}^4x \;\mathrm{tr}\, G_{\mu\nu}\widetilde{G}^{\mu\nu}$$

The θ-Term Has Physical Consequences

$$\theta G_{\mu\nu}\tilde{G}^{\mu\nu} = \partial_\mu \left[\epsilon^{\mu\nu\rho\sigma} A^a_\nu \left(G^a_{\rho\sigma} - \frac{g}{3} f^{abc} A^b_\rho A^c_\sigma \right) \right]$$

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Naively, this appears like it should have no physical effects, but it *does* appear in real observables. For one, note that under a chiral rotation, due to the anomaly:

$$u \to e^{i\alpha} u, \quad d \to e^{i\alpha} d, \quad \theta \to \theta - 2\alpha$$

Therefore there is an *invariant* angle:

$$\bar{\theta} \equiv \theta + \arg\det m_u + \arg\det m_d$$

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Therefore there is an *invariant* angle:

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This manifests both in the vacuum structure of QCD, but also appears when matched onto the chiral Lagrangian, e.g., in the CP-odd pion-nucleon interactions:

$$\mathcal{L} \supset \bar{g}^{(0)}_{\pi NN} \bar{N} \tau^a N \pi^a + \ldots$$

The Strong CP Problem

Most significantly, $\bar{\theta}$ gives rise to a neutron electric dipole moment:

$$\mathcal{L} \supset i d_N F_{\mu\nu} \bar{N} \gamma^{\mu\nu} \gamma_5 N$$

Where the dipole moment is given by:

(see Anson Hook's TASI notes, [1812.02669] for a pedagogical calculation)

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WHY? We just argued that $\bar{\theta}$ is just a phase!
This is the strong CP problem.

Two Classes of Solutions to the Strong CP Problem:

- Introduce an anomalous $U(1)_{\mathrm{PQ}}$ symmetry.
    - Predicts a light goldstone boson — the axion, which has a rich & well-studied Cosmology

Two Classes of Solutions to the Strong CP Problem:

- Introduce an anomalous $U(1)_{\rm PQ}$ symmetry.
  - Predicts a light goldstone boson — the axion, which has a rich & well-studied Cosmology
- Assume CP (or P) is an *exact* spacetime symmetry, which is spontaneously broken
  - Does not require new light physics, but has other interesting consequences which are less explored

CP as an Exact Symmetry

Is CP an Exact Symmetry of Nature?

Some hints in the SM:

- CP Violation only observed in flavor-changing processes.
- Running of $\bar{\theta}$ arises only at seven loops in the SM.
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Moreover, there are arguments that (3+1)-dimensional CP can arise from the spacetime symmetries of superstring theory (e.g., Strominger-Witten, 1985)
(c.f. Dine, Leigh, MacIntire, 1992 and Choi, Kaplan, Nelson, 1993)

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The challenge: spontaneously break CP, generating an $\mathcal{O}(1)$ CKM phase, not suppressed by powers of v/Λ_{CP}

$\Longrightarrow$ Solved by Ann Nelson in 1984

Parity/CP in a Fundamental Theory

Parity is usually introduced as the symmetry $(t, \vec{x}) \mapsto (t, -\vec{x})$,
$\Longrightarrow$ this is clearly insufficient for a theory of (quantum) gravity.

Instead, say a theory has an exact parity symmetry if it can be defined on *non-orientable manifolds*.
(CP is simply parity along with an internal symmetry)

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In quantum gravity, parity can be a *gauge* symmetry, where we sum over both orientable and non-orientable manifolds in the path integral.

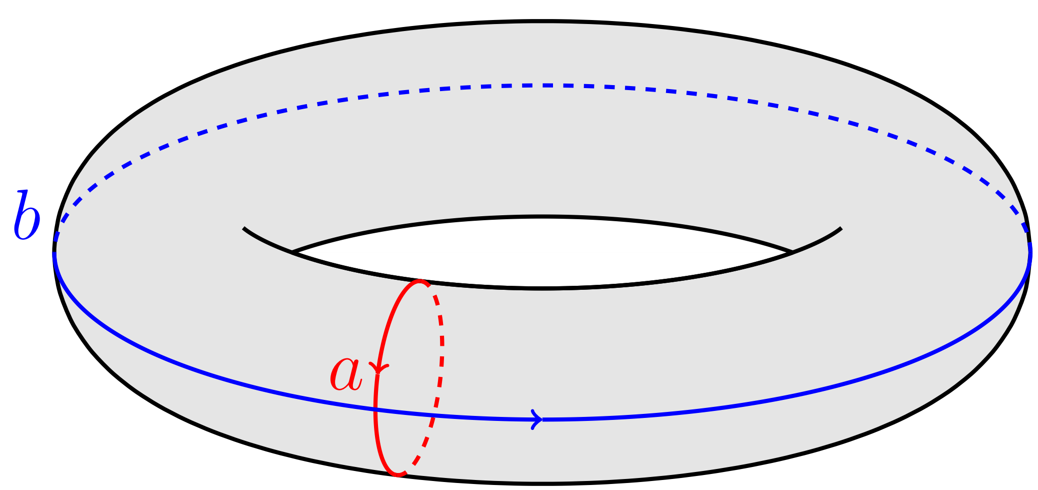
Easiest to understand via analogy to discrete *internal* symmetries with background “gauge” fields…

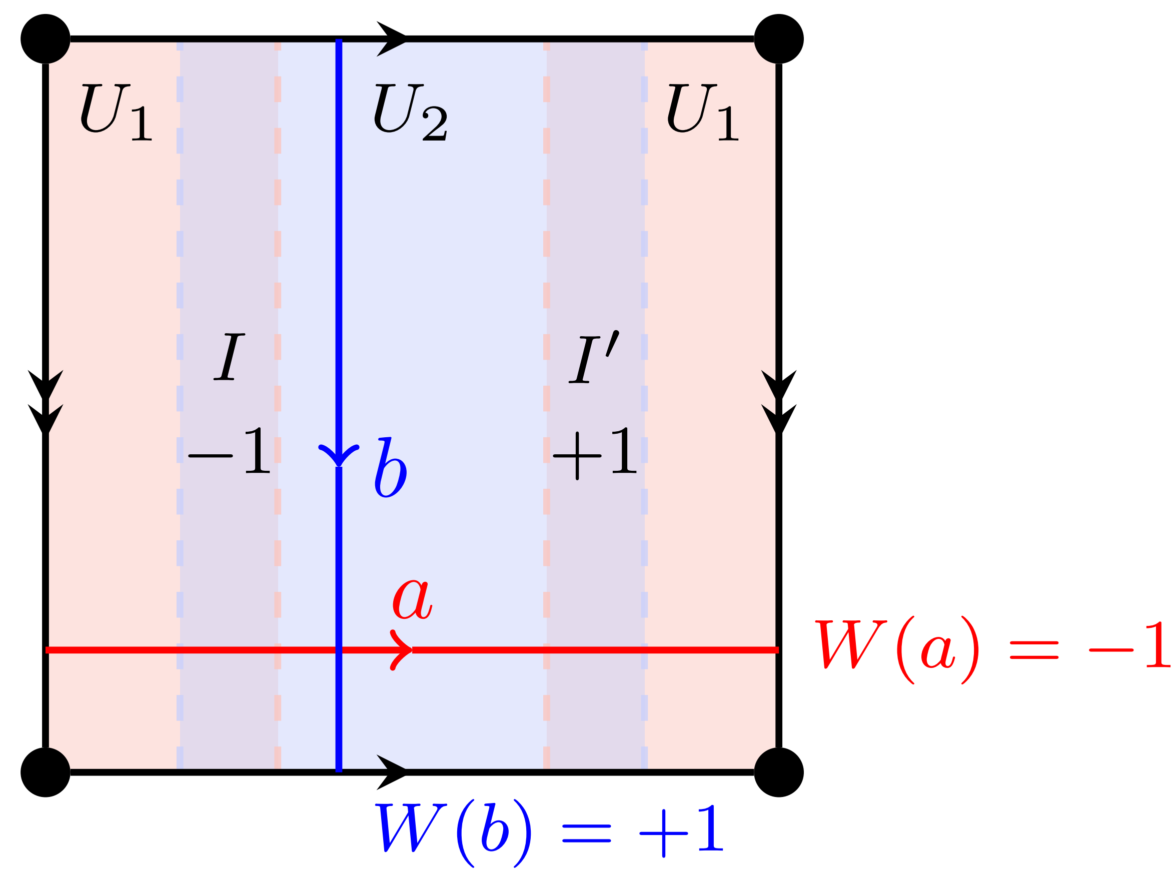
Discrete Internal Symmetries

Useful to discuss “gauge configurations” in terms of holonomies around cycles in spacetime

Or, equivalently, in terms of transition functions (elements of *G*) on overlaps of coordinate patches

Example: $G = \mathbb{Z}_2$ on the torus



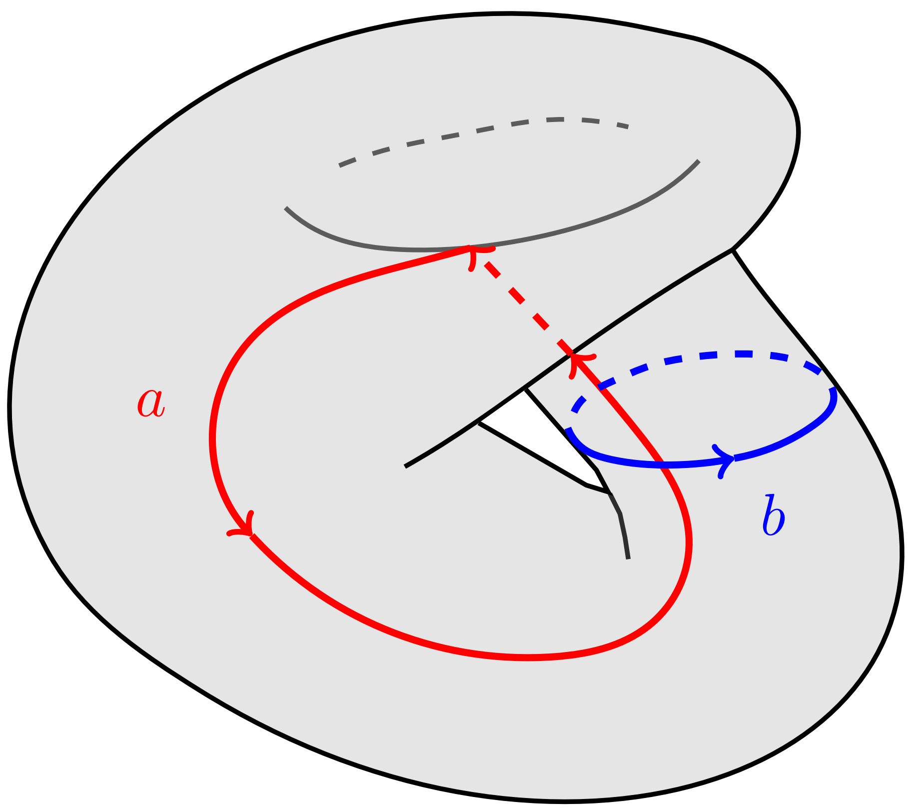


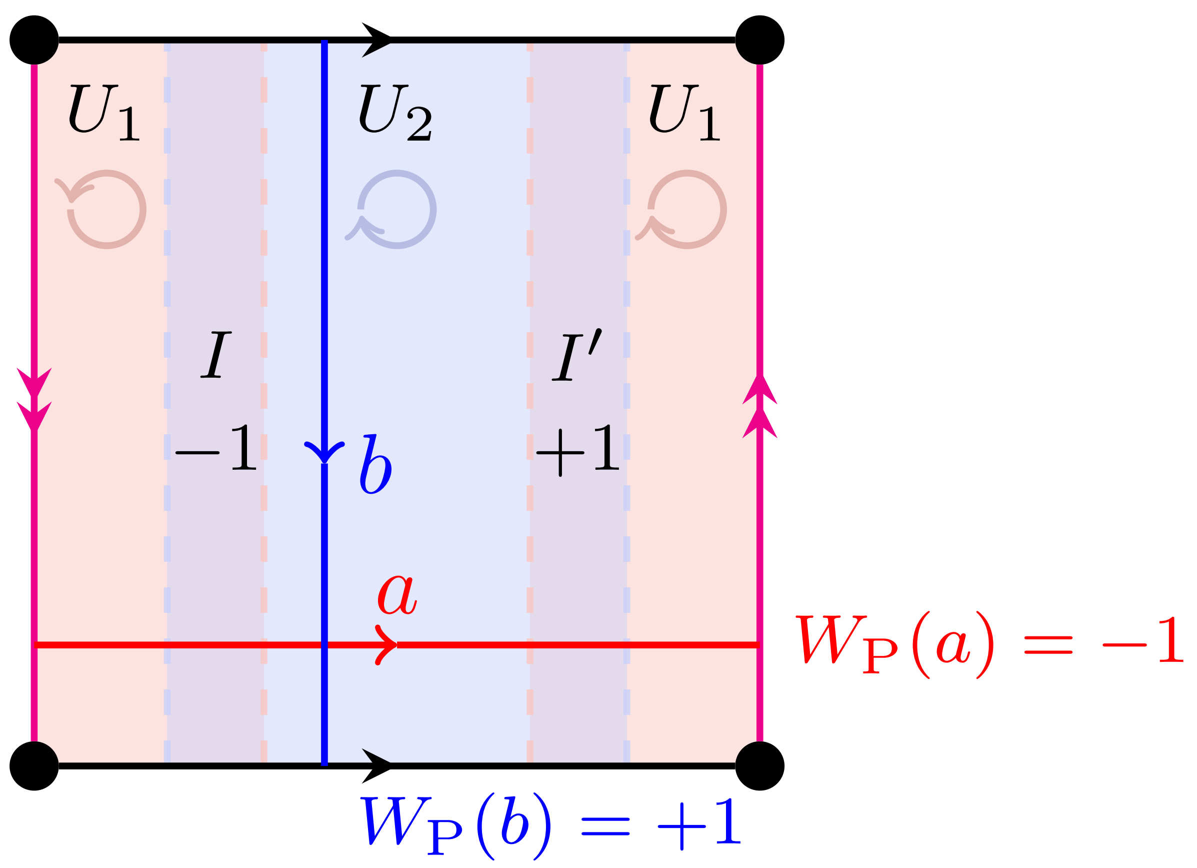
Figures borrowed from McNamara, Reece [2212.00039]

Discrete *Spacetime* Symmetries

Example: Klein Bottle

Parity as a global $\mathbb{Z}_2^{\mathrm{P}}$ symmetry works the same way, but now the background "parity gauge field" is *fixed* by the choice of background manifold





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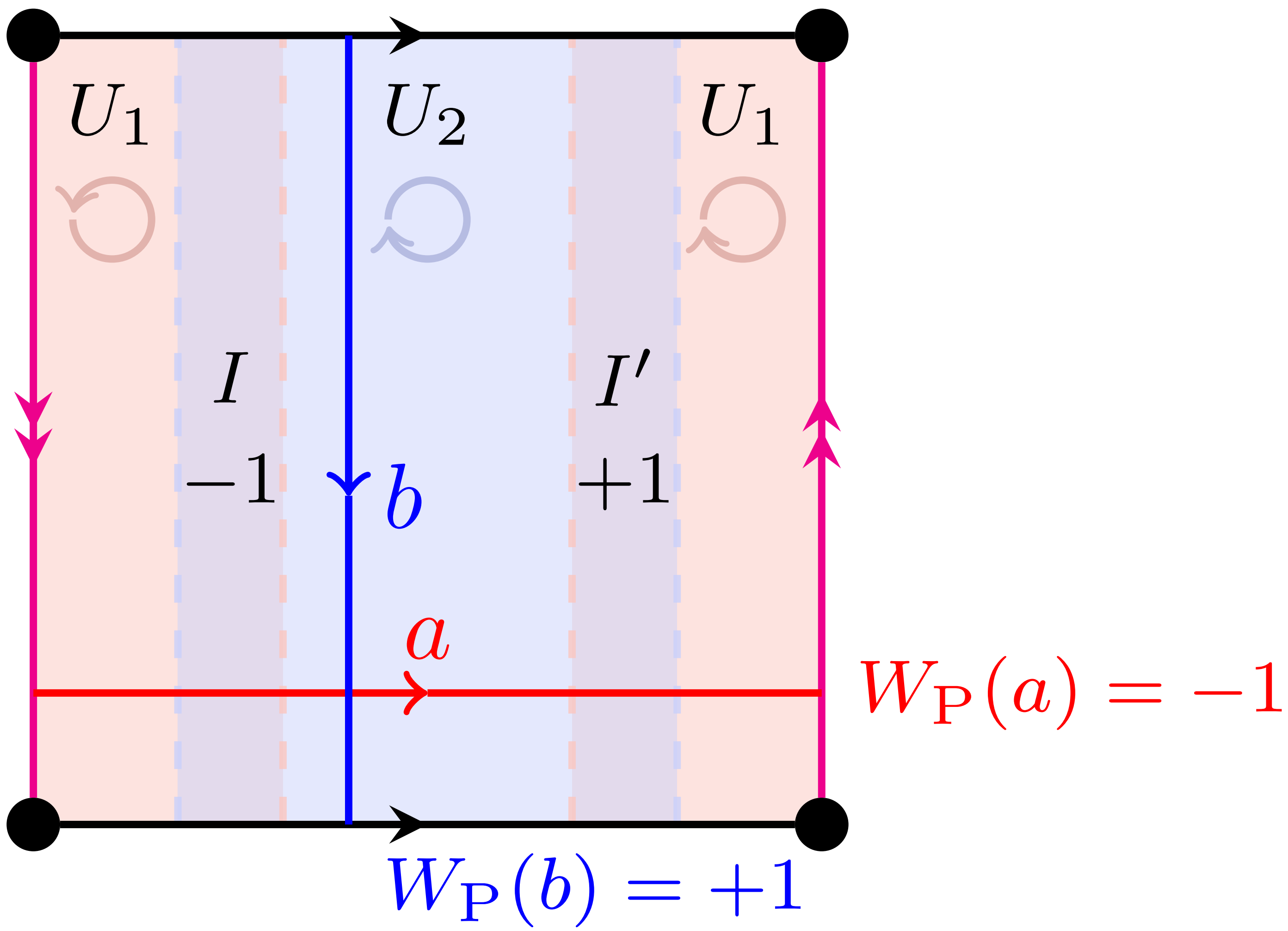
Parity Transition Functions



As with internal symmetries, we work in different coordinate patches and impose gluing rules on overlaps.

Pseudotensors (e.g., pseudoscalars) pick up a minus sign when moving to patches with opposite orientation.

This extends to fermions (with CP) in the obvious way.

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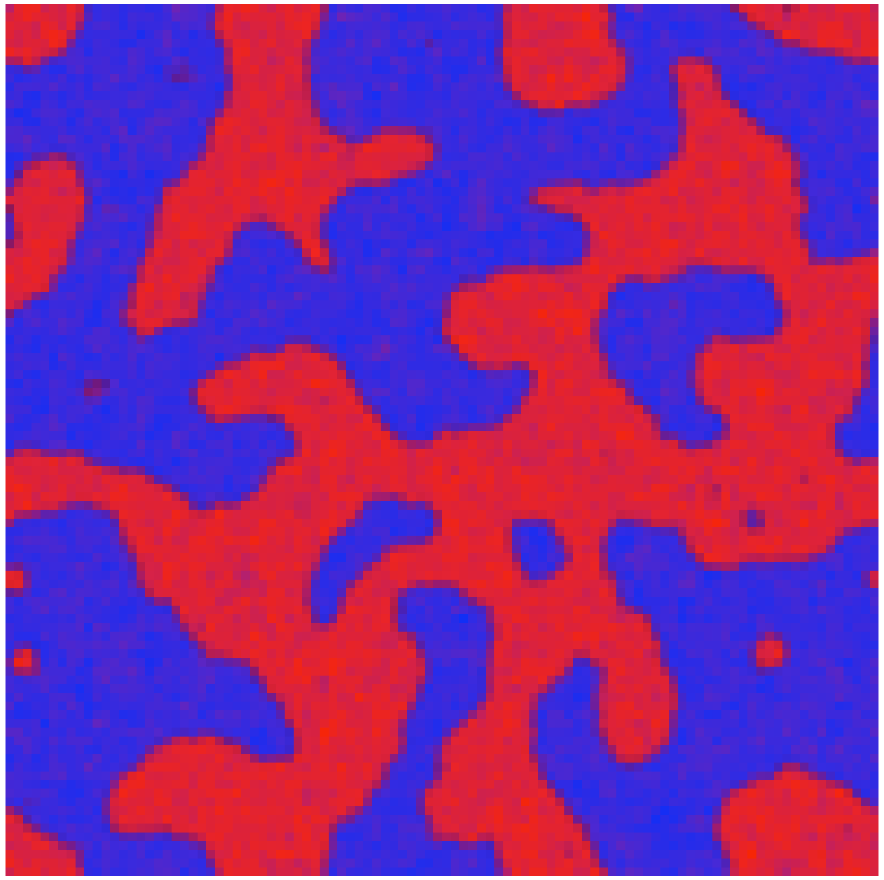
Domain Wall Problems in Cosmology

Domain walls generically form whenever a discrete symmetry (including P, CP) is spontaneously broken

For an exact global symmetry, these domain walls are *exactly stable*



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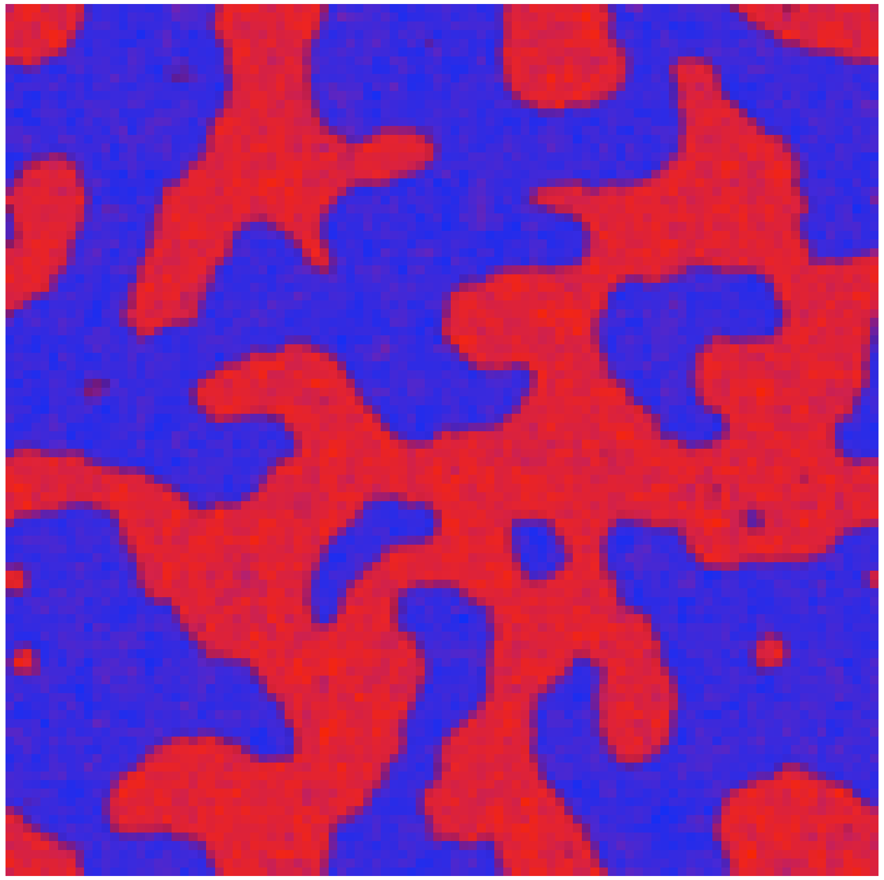


[1010.2328]

Energy density of domain walls redshifts as

$$\rho \sim a^{-1}(t)$$

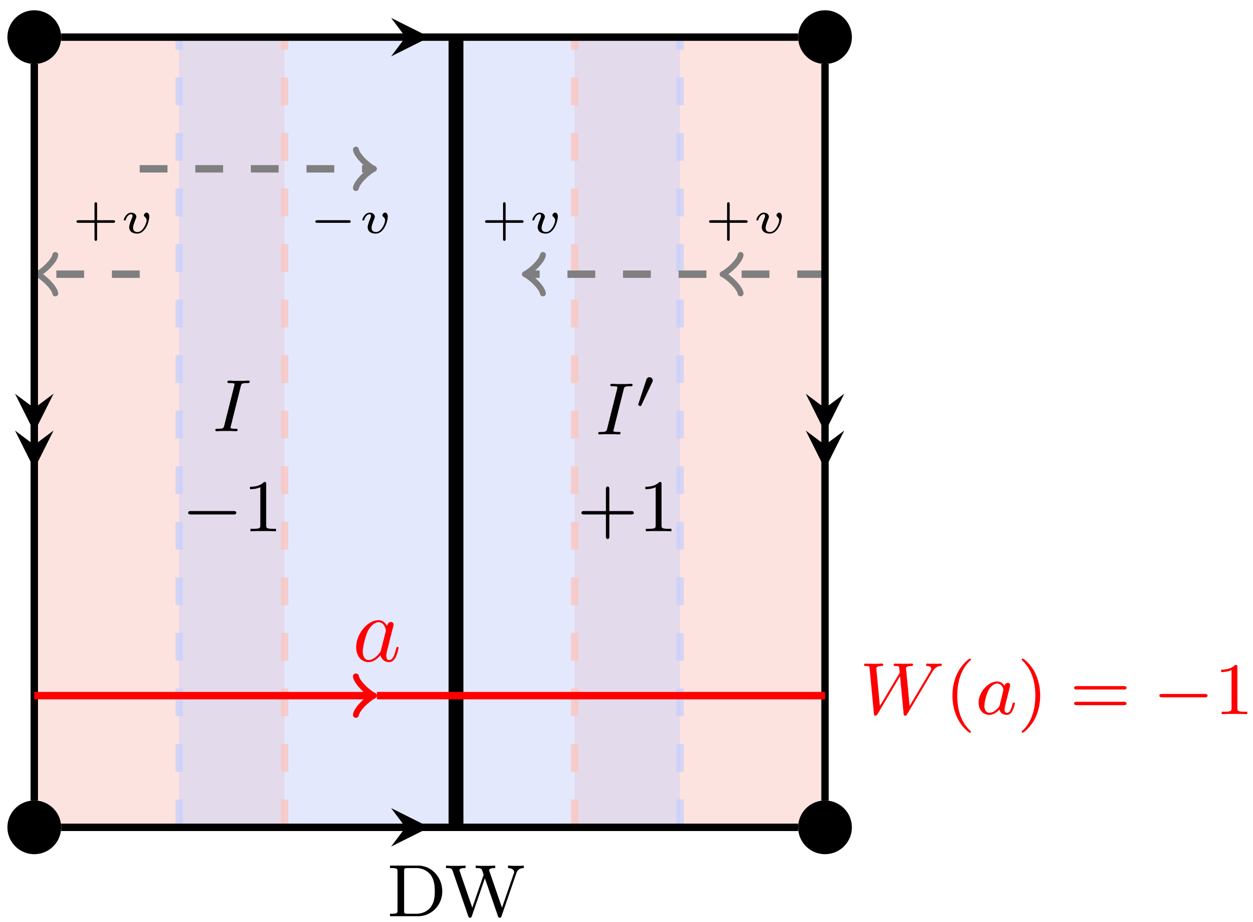
$\Longrightarrow$ very quickly dominates over matter (a^{-3}) and radiation (a^{-4})

Must be inflated away or dynamically destroyed to recover viable cosmology

Topological Stability of Domain Walls

First, for a *global internal* symmetry

Consider a $\mathbb{Z}_2$-odd field ϕ with $\langle \phi \rangle = \pm v$, which spontaneously breaks the $\mathbb{Z}_2$ global symmetry. *In the presence of our background field* there is a domain wall as a **topological requirement**.



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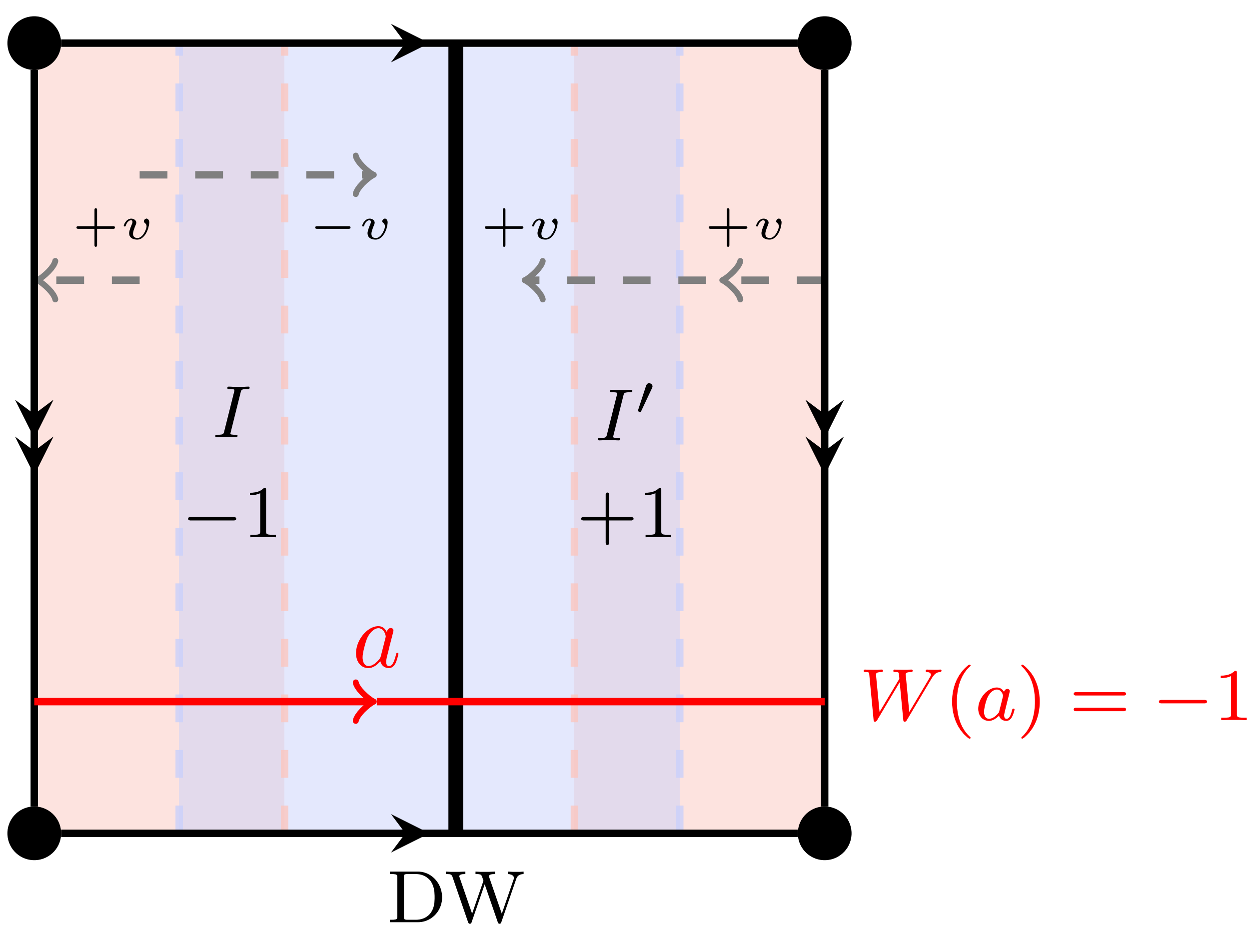


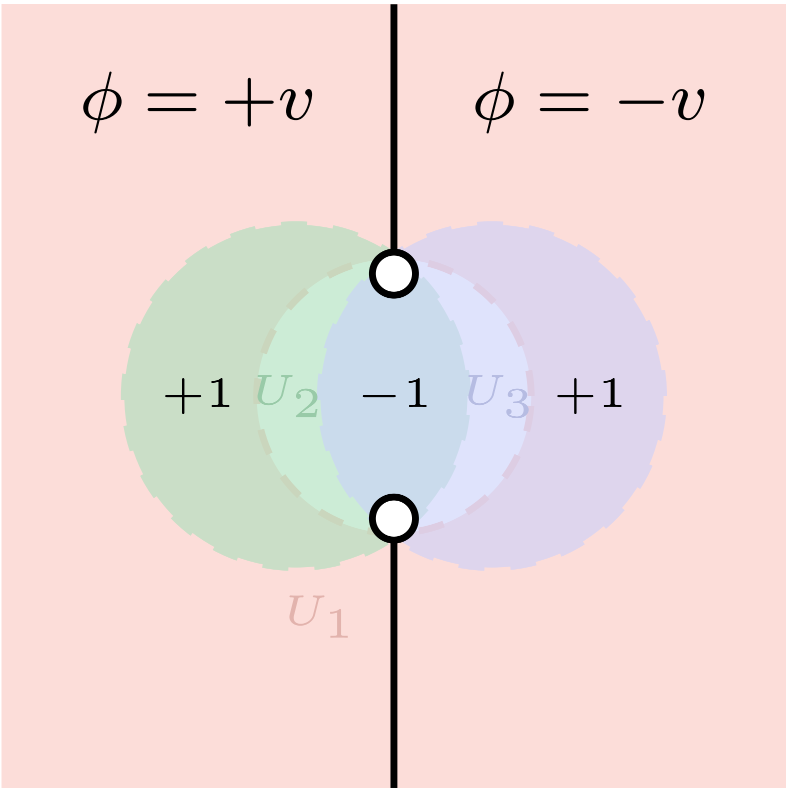
This proves the domain wall is stable, and that no local process can tear a hole in it.

But since no local process is allowed, it must be stable on *any background!*

How Can Domain Walls Be Destabilized?

If the symmetry is *explicitly* broken, domain walls are no longer stable.

If the symmetry is *gauged* there exist *dynamical vortices* (or strings), which insert a twist (gauge transformation)
$\Longrightarrow$ allows domain wall to *end!*



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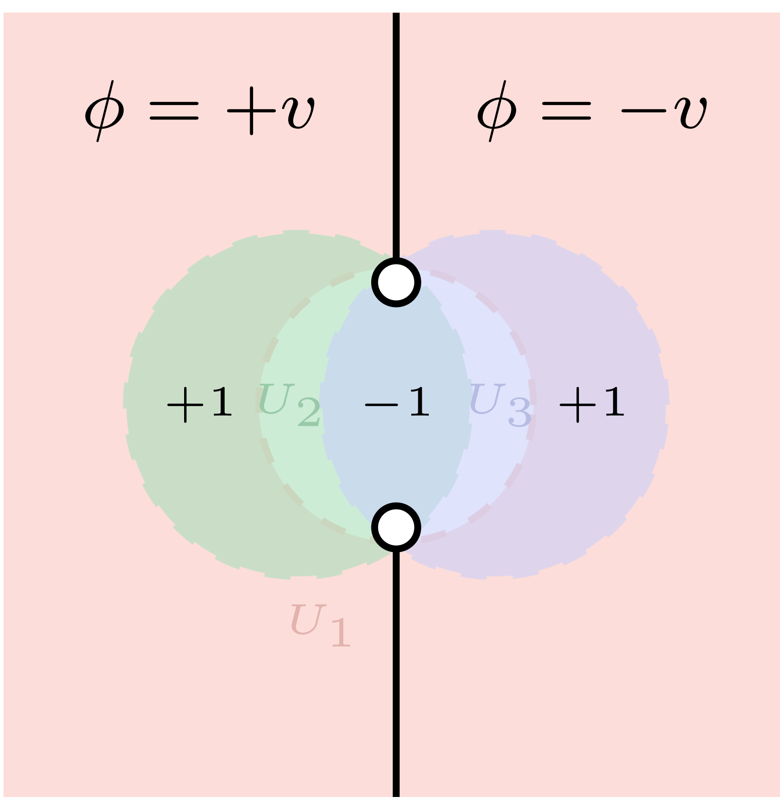

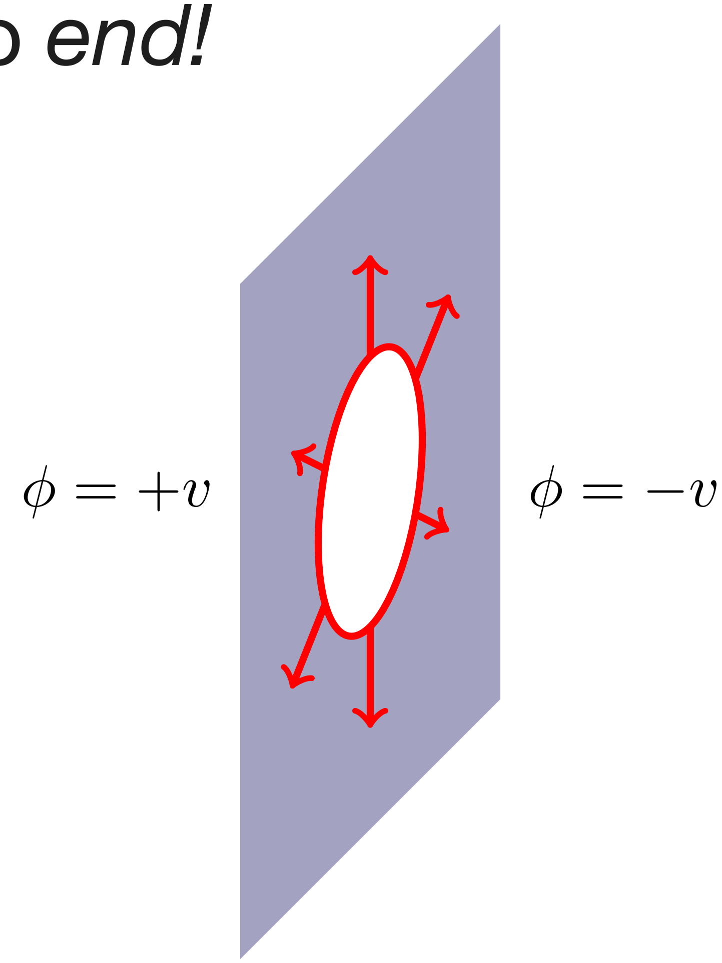

Holes can spontaneously nucleate and eat up walls, or collapse network from ends

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Parity/CP Domain Walls are *Exactly Stable*

J. McNamara and M. Reece, [`arXiv:2212.00039`]

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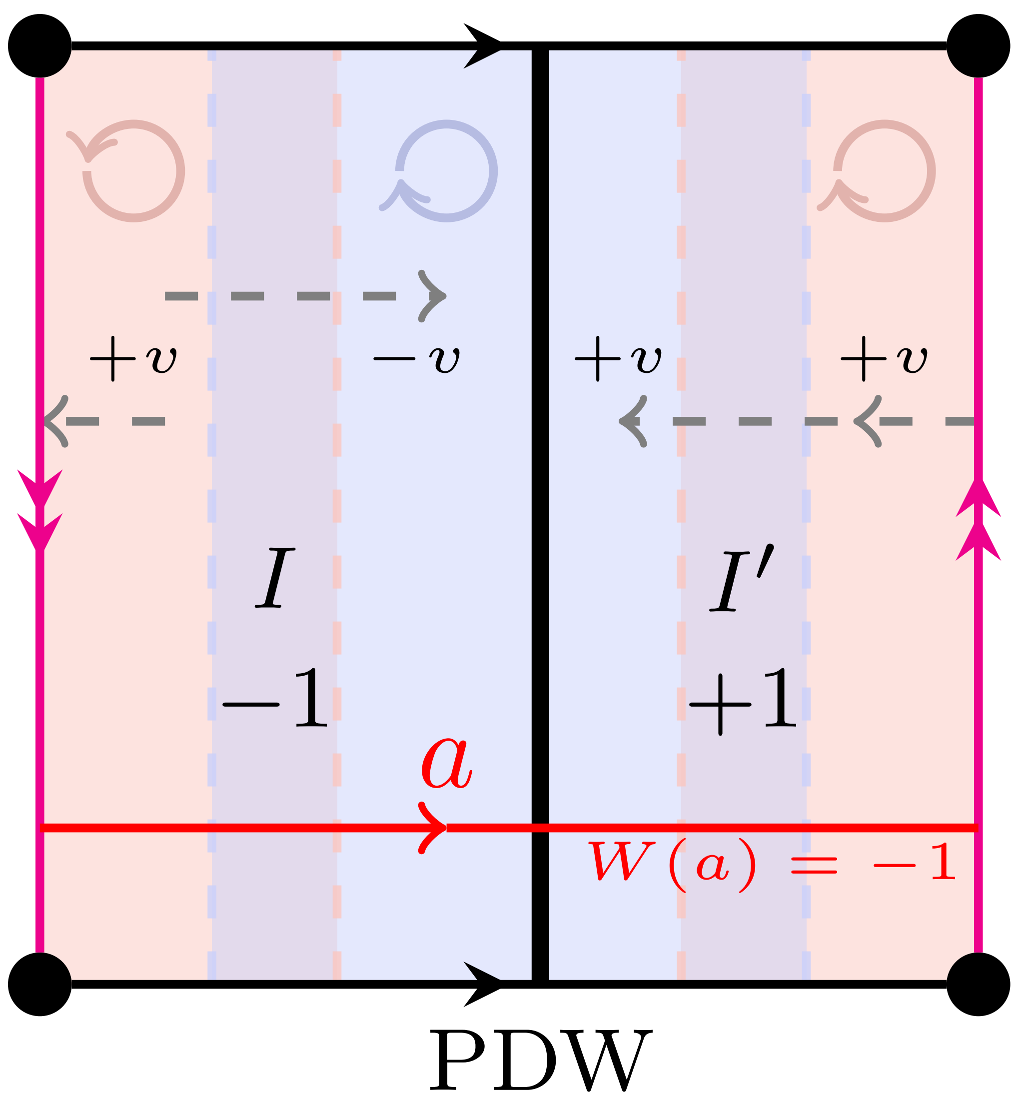

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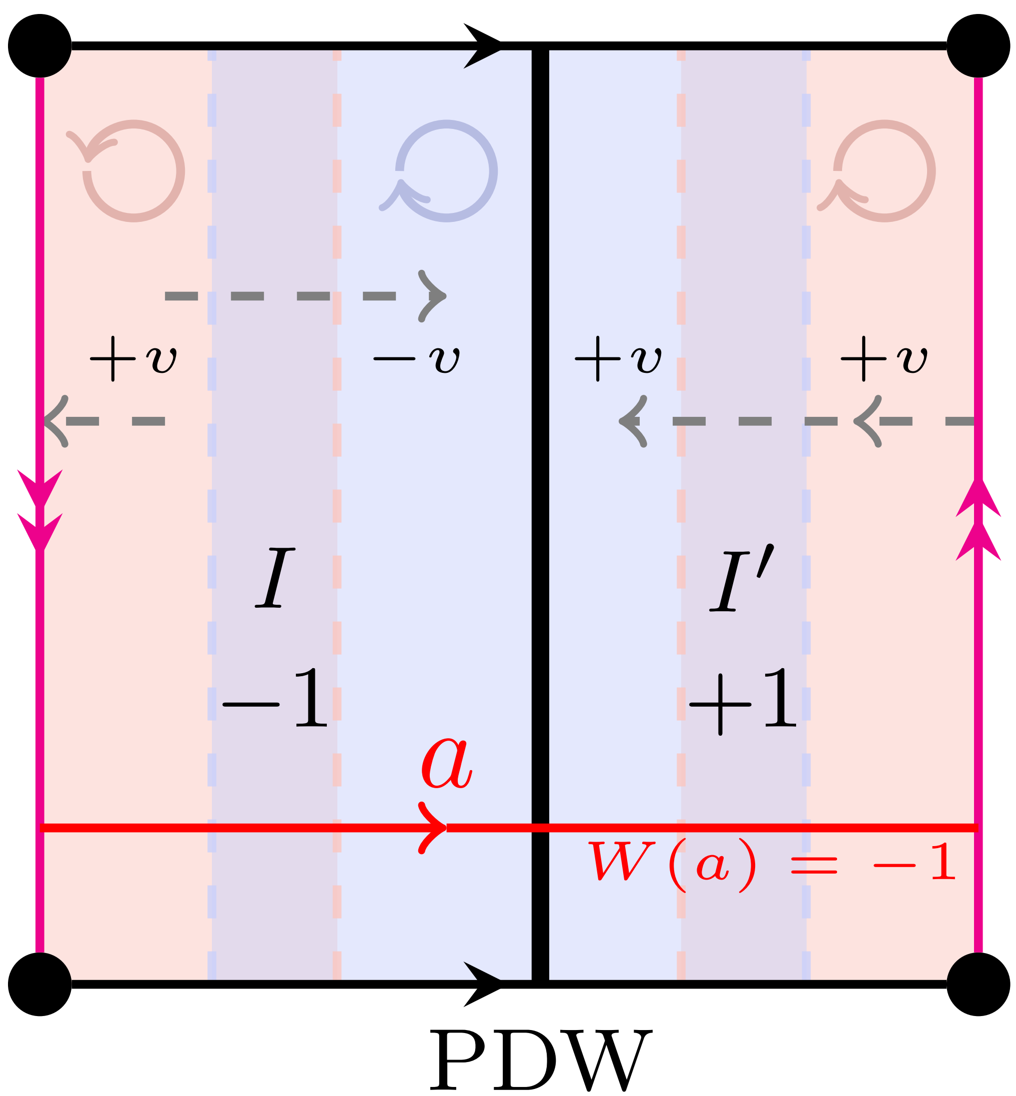

For a *gauged* parity symmetry, need to ask if there exist "parity vortices", which change the boundary conditions.

McNamara & Reece proved that such an object *cannot exist*

(Key fact: all closed 1-manifolds are orientable)

Parity (or CP) domain walls *must* be inflated away!

The Nelson-Barr Quality Problem & Cosmological Consequences

A Minimal Nelson-Barr Model

Bento, Branco, Parada (BBP)

Extend the SM with a pair of vector-like quarks, $D, \bar{D}$ transforming like the down quark, and N pseudoscalars, η_a

Assume CP symmetry, and impose a Z_N symmetry:

$$\eta_a \to \mathrm{e}^{2\pi i k/N} \eta_a , \quad D \to \mathrm{e}^{-2\pi i k/N} D, \quad \bar{D} \to \mathrm{e}^{2\pi i k/N} \bar{D}$$

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The down-type mass, Yukawa terms are:

$$\mathcal{L} \supset \mu_D D\bar{D} + (\lambda_d)^i{}_j Q_i H^c \bar{d}^j - f_i^a \eta_a D\bar{d}^i + \mathrm{h.c.}$$

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Note: $Q_i H^c \bar{D}$ term is forbidden by Z_N

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This solves the strong CP problem at tree-level:

$$\mathcal{L} \supset \begin{pmatrix} Q & D \end{pmatrix} \underbrace{\begin{pmatrix} \lambda_d v/\sqrt{2} & 0 \\ \sum_a f_i^a \langle \eta_a \rangle & \mu_D \end{pmatrix}}_{\mathcal{M}_0} \begin{pmatrix} \bar{d} \\ \bar{D} \end{pmatrix}$$

Vanishes by CP symmetry

$$\Longrightarrow \bar{\theta} = \theta_0 + \arg\det \mathcal{M}_0 = 0$$

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Vanishes by CP symmetry

$$\implies \bar{\theta} = \cancel{\theta_0} + \arg\det\mathcal{M}_0 = 0$$

And the effective mass matrix for the SM quarks is

$$(m_0^2)^i_j = (m_d)^i_k \underbrace{\left(\delta^k_l + \frac{F^{\dagger\,k} F_l}{F_p F^{\dagger\,p} + \mu_D^2}\right)}_{\text{An } \mathcal{O}(1) \text{ complex phase!}} (m_d^T)^l_j$$

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Warning: this minimal example requires some fine-tuning:

$$\mathcal{L} \supset \underbrace{\lambda_{ab}\eta_a\eta_b^\dagger H^\dagger H}_{\text{Correction to the Higgs mass } \propto \Lambda_{\rm CP}^2} + \gamma_{abcd}\eta_a\eta_b^\dagger\eta_c\eta_d^\dagger$$

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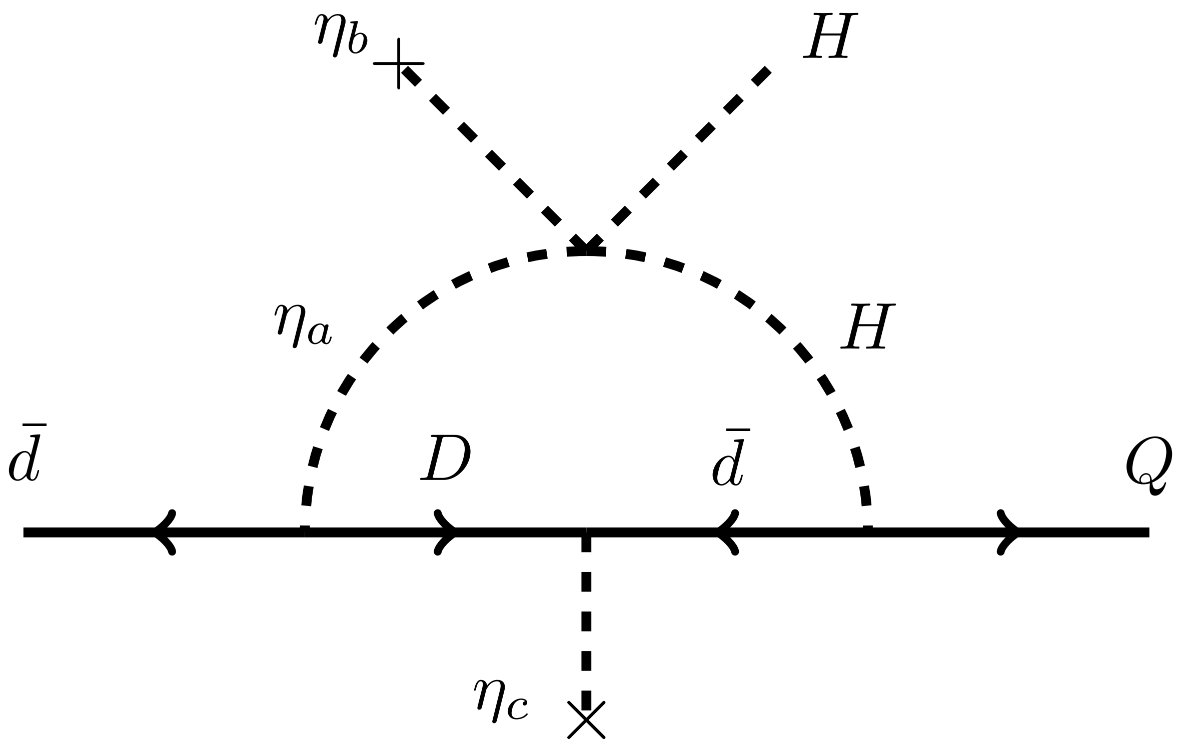
Correction to the Higgs mass $\propto \Lambda_{\rm CP}^2$

Also corrections to $\bar{\theta}$ at one-loop:

$$\Delta\bar{\theta} \sim \frac{1}{8\pi^2} f_k f_k \lambda \frac{\Lambda_{\rm CP}^2}{m_\eta^2} \log\left(\frac{v^2}{\Lambda_{\rm CP}^2}\right)$$



These could be ameliorated e.g., with supersymmetry

(see Dine, Draper [1506.05433])

The Nelson-Barr "Quality Problem"

Generically, $\bar{\theta}$ will receive corrections from non-renormalizable operators. In our setup, these include

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Naively contribute $\Delta\bar{\theta} \sim \dfrac{\Lambda_{\rm CP}}{\Lambda_{\rm EFT}}$,

This sets an ***upper bound*** on the scale of CP breaking

$$\Lambda_{\rm EFT} = M_{\rm Pl} \implies \Lambda_{\rm CP} \lesssim 10^8\,{\rm GeV}$$

An Aside: More Careful Estimates Involve Flavor

$$\mathcal{L} \supset \frac{h_a^i}{\Lambda_{\mathrm{EFT}}} \eta_a^\dagger Q_i H^c \bar{D} \qquad \Longrightarrow \qquad \Delta\bar{\theta} \sim \frac{\langle \eta_a \rangle \langle \eta_b \rangle}{\mu_D \Lambda_{\mathrm{EFT}}} \left(f_i^b (\lambda_d^{-1})^i{}_j h_a^j \right)$$

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We can treat the product $f_i^b \, h_a^j$ as a spurion of the SM flavor group:

- "Minimal Flavor Violation": $f_i^b (\lambda_d^{-1})^i{}_j \, h_a^j \sim 1,$
  recover $\Lambda_{\rm CP} \lesssim 10^8$ GeV bound
- New flavor-aligned spurion, $f_i^b (\lambda_d^{-1})^i{}_j \, h_a^j \lesssim 1/y_d \approx 10^5,$
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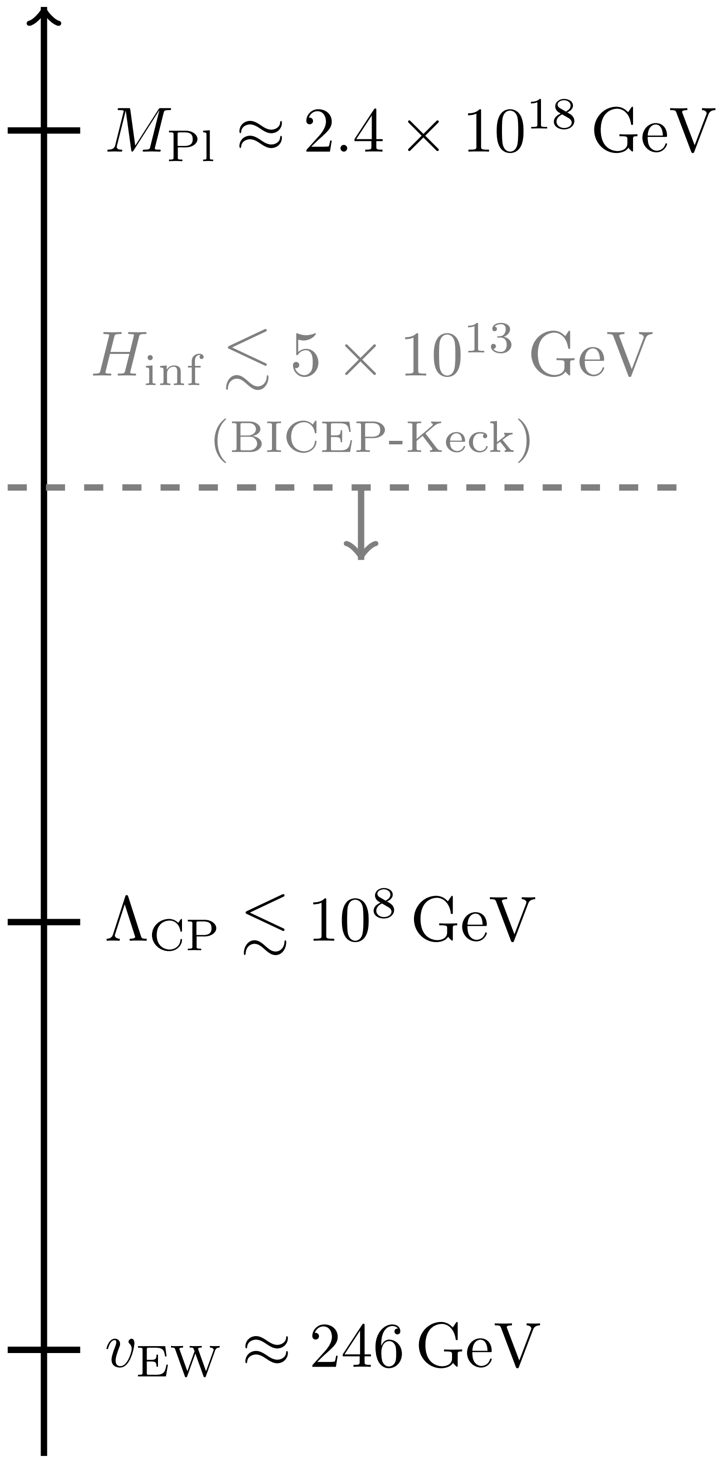
See e.g., Egaña-Ugrinovic, SDH, Meade, 1811.00017, 1908.11376 for an example

“Late”-Time Spontaneous CP Breaking?

What if we take the $\Lambda_{CP} \lesssim 10^8$ GeV bound seriously?



Inflation has to occur *after* Spontaneous CP breaking

$$H_{inf} \simeq T_{reh} < \Lambda_{CP} \lesssim 10^8 \text{ GeV}$$

Constrains a lot of potential dynamics / signatures in the early universe!

(Assuming single field inflation where reheating proceeds via perturbative decay of the inflation, for concreteness, but most of these lessons are general)

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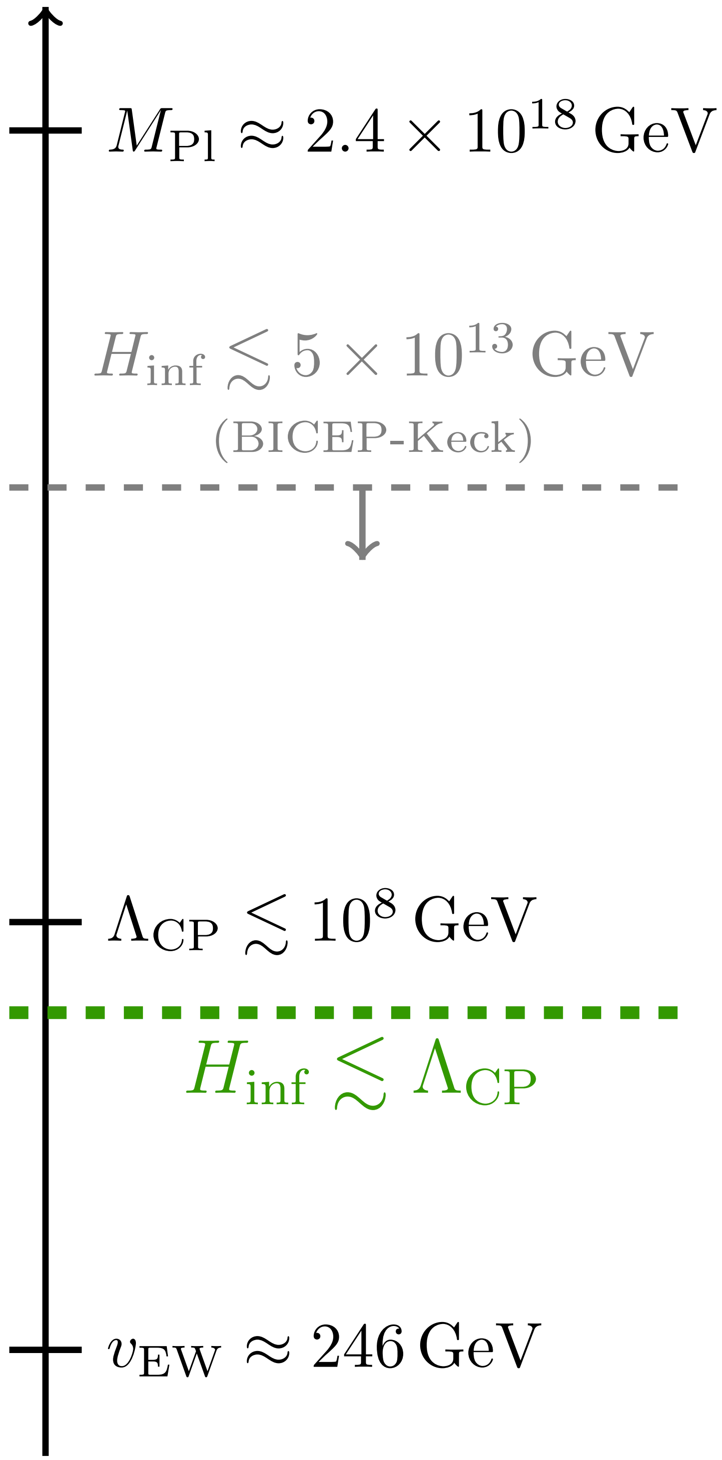

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Implications for Inflationary Cosmology

We can quantify this bound on $H_{\rm inf}$ a bit more

Tensor modes in the CMB are the “smoking gun” of inflation.

For single-field inflation, the tensor-to-scalar ratio directly measures $H_{\rm inf}$:

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} \approx 9.65 \times 10^7 \frac{H_{\rm inf}^2}{M_{\rm Pl}^2}$$

The current bound from BICEP/Keck is $r < 0.036$, which sets $H_{\rm inf} \lesssim 5 \times 10^{13}$ GeV. [2110.00483]

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In contrast, $H_{\rm inf} \leq \Lambda_{\rm CP}$ implies

$$r \leq 1.7 \times 10^{-13} \left(\frac{\Lambda_{\rm CP}}{10^8 \, {\rm GeV}} \right)^2$$

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This is also a challenge for model-building. For instance, the slow roll parameter:

$$\epsilon \equiv \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

is related to r in single-field inflation by a constant factor, $r = 16\epsilon$.

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The Lyth bound also relates r to the total inflaton field evolution during inflation,

$$\frac{\Delta\phi}{M_{\rm Pl}} \lesssim 10^{-6}$$

i.e., we require “small field inflation”.

Model building challenges!

Initial condition problems, etc.

Implications for High-Scale Baryogenesis

With some reasonable assumptions about reheating, we also have $T_{\mathrm{reh}} < H_{\mathrm{inf}} \lesssim \Lambda_{\mathrm{CP}}$.

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Davidson & Ibarra noted that this leads to a bound: [hep-ph/0202239]

$$\varepsilon \lesssim \frac{3}{8\pi}\frac{M_N m_\nu}{v^2} \quad \underset{T_{\rm reh} \geq M_N}{\Longrightarrow} \quad T_{\rm reh} \gtrsim 10^{8-10}\,\mathrm{GeV}$$

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Challenging to have N thermalize!

Is There Another Way?

Two ways to evade the $H_{\text{inf}}, T_{\text{reh}} \lesssim 10^8$ GeV constraint:

- Sequester the CP violation in a hidden sector
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Sequestered Spontaneous CP Breaking?

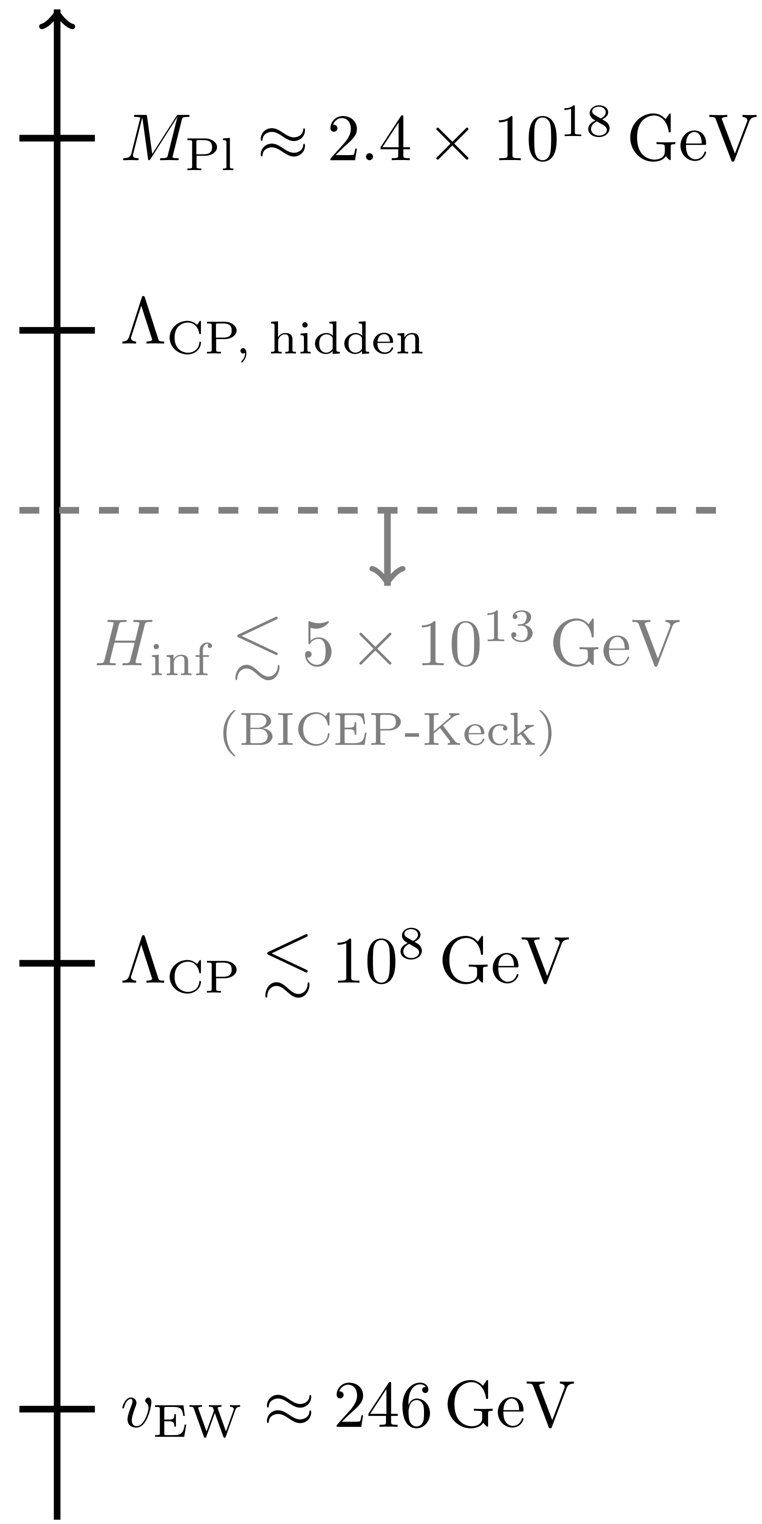
Another possibility: break CP first in a sequestered hidden sector ($\Lambda_{CP} \lesssim 10^8$ GeV constraint doesn't apply), then inflate away the stable domain walls

Subsequently, small interactions with the visible sector make *effective* explicit CPV

$\Longrightarrow$ visible sector CP breaking gives *unstable* domain walls

Challenges:

- Keep explicit $\bar{\theta}$ term small enough?
- Short enough lifetime for Dos?
- Gravitational Wave signatures?



Chiral Nelson-Barr Models

One way to ameliorate the quality problem is to forbid the dimension-5 operators,

$$\frac{1}{\Lambda_{\text{EFT}}}\eta_a^\dagger \eta_b D\bar{D}, \qquad \frac{1}{\Lambda_{\text{EFT}}}\eta_a^\dagger Q_i H^c \bar{D}$$

by taking $D, \bar{D}$ to transform *chirally* under a new symmetry, $U(1)_X$.

See also Valenti, Vecchi [2106.09108]

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by taking $D, \bar{D}$ to transform *chirally* under a new symmetry, $U(1)_X$.

We must introduce a new scalar, ρ to give mass to $D, \bar{D}$,

$$\mathcal{L} \supset -y_D \rho D\bar{D}, \qquad \mu_D = y_D \langle \rho \rangle$$

The rest of the analysis of the NB solution is precisely the same as our minimal model, with the $U(1)_X$ replacing the necessary Z_N symmetry.

See also Valenti, Vecchi [2106.09108]

What about Anomalies?

We can cancel *all* mixed anomalies with only one additional set of vector-like fermions, $B, \bar{B}$, where charge assignments are

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$
D	$\mathbf{3}$	–	$-1/3$	-1
$\bar{D}$	$\bar{\mathbf{3}}$	–	$+1/3$	-5
B	$\mathbf{3}$	–	$+1/3$	$+1$
$\bar{B}$	$\bar{\mathbf{3}}$	–	$-1/3$	$+5$
ρ	–	–	0	$+6$
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This works because we have taken $U(1)_X$ to be a linear combination of hypercharge and $B - L$, which is always anomaly free for the SM fields.

$$X = -4Y - (B - L)$$

Chiral Nelson-Barr Models

No additional renormalizable operators other than

$$\mathcal{L} \supset -y_D \rho D\bar{D} - y_B \rho^\dagger B\bar{B} + \mathrm{h.\,c.}$$

All possible dimension-5 operators are forbidden; the quality problem arises again at dimension-6:

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These contribute (assuming Minimal Flavor Violation)

$$\Delta\bar{\theta} \simeq \frac{1}{y_D}\frac{\Lambda_{\rm CP}^2}{\Lambda_{\rm EFT}^2} \quad \Longrightarrow \quad \Lambda_{\rm CP} \lesssim 10^{13}\ \mathrm{GeV}$$

High enough to recover most of the Cosmology we're interested in!

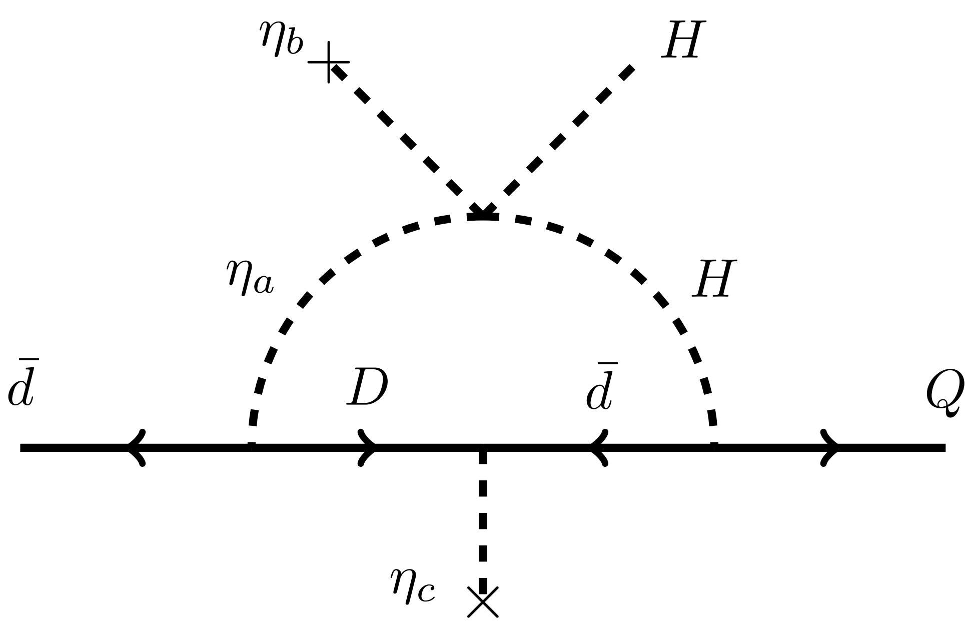
Conclusions

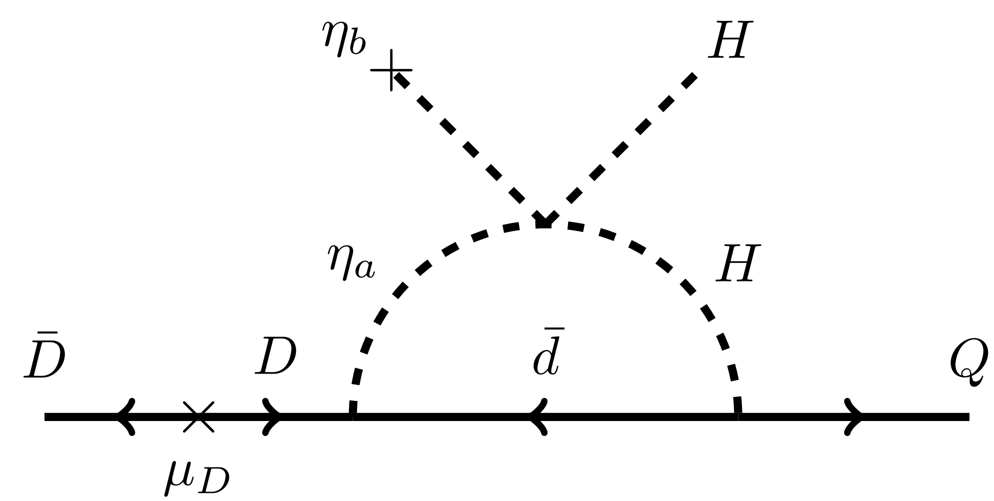
- An exact CP symmetry can solve the Strong CP problem, but leads to deep questions about the nature of such a symmetry.
- Spontaneous breaking of CP leads to *exactly stable* domain walls, which must be inflated away.
- Nelson-Barr “Quality Problem”: The scale of spontaneous CP breaking cannot be too large — these facts together constrain our cosmological history.
- “Chiral Nelson-Barr models” are one way of alleviating the quality problem — many open model-building challenges remain!

Backup: Radiative Corrections to $\bar{\theta}$

$$\Delta\bar{\theta} = \arg\det\left(\mathcal{M}_0 + \mathcal{M}_1\right) \simeq \arg\left(1 + \mathrm{tr}\left(\mathcal{M}_0^{-1}\mathcal{M}_1\right)\right)$$

$$\simeq \mathrm{Im}\left(m_d^{-1}\mathcal{M}^{(1)}_{Q\bar{d}} - \frac{1}{\mu_D}\left(F m_d^{-1}\mathcal{M}^{(1)}_{Q\bar{D}} + \mathcal{M}^{(1)}_{D\bar{D}}\right)\right)$$





$$\Delta\bar{\theta} \sim \frac{1}{8\pi^2} f_k f_k \lambda \frac{\Lambda_{\mathrm{CP}}^2}{m_\eta^2} \log\left(\frac{v^2}{\Lambda_{\mathrm{CP}}^2}\right)$$

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This can be suppressed if λ is kept small by e.g., supersymmetry (which also protects the Higgs mass).

In supersymmetry, λ only arises from coupling to SUSY breaking, and is of order $|F|^2/M_{\rm Pl}^4$, where $\sqrt{F}$ is the SUSY breaking scale.

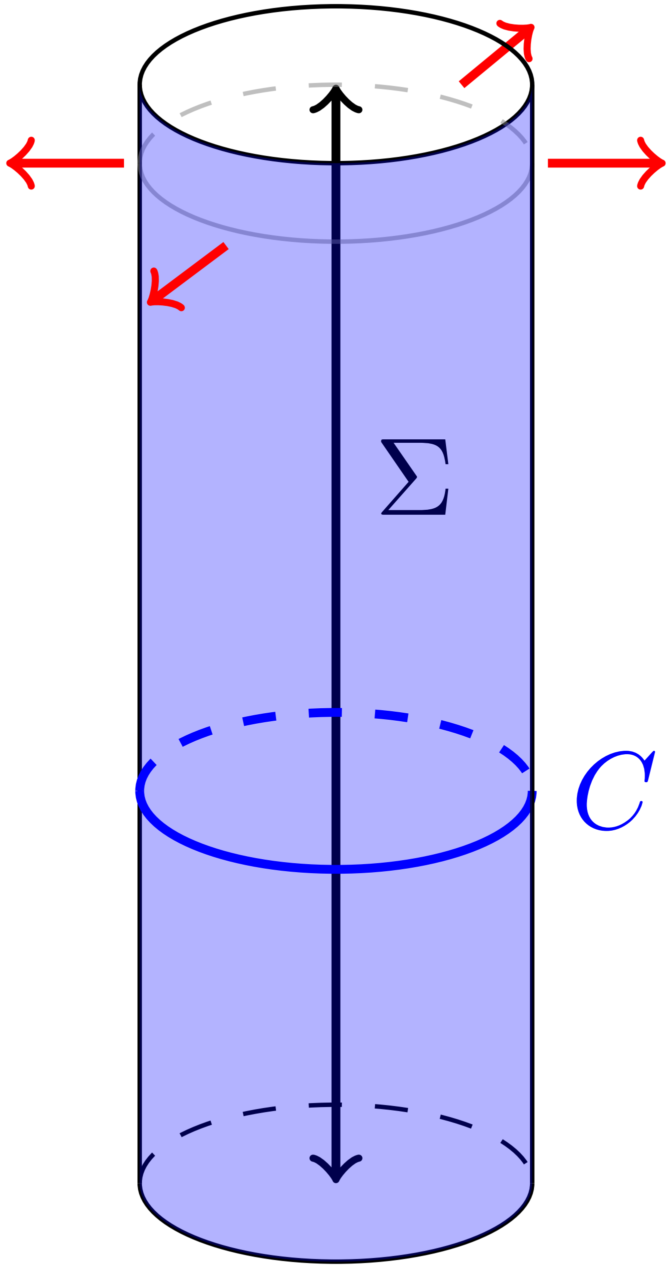
The one-loop correction to the quartic scales as

$$\delta\lambda_{ab} \sim \frac{1}{16\pi^2} f_i^a f_j^b (\lambda_d)^k{}_i (\lambda_d)^k{}_j \log\frac{\Lambda_{\rm UV}^2}{\Lambda_{\rm CP}^2}$$

Which is consistent for $f \sim 0.1$, $\lambda_d \sim y_b$, and $\lambda_{ab} \sim 10^{-7}$

Backup: Codimension-2 Boundary Conditions

(Borrowed from M. Reece)



A "parity vortex" would be a codimension-2 boundary condition in the (semiclassical) quantum gravity path integral.

For gauged parity, we sum over spacetimes X, with $\partial X = \Sigma \times C$, with **fixed** Σ; Want a parity Wilson line $W_P(C) = -1$. But all closed 1-manifolds are orientable, so no such boundary condition exists.

$\Longrightarrow$ an *even* number of domain walls ends on any codimension-2 object.

$\Longrightarrow$ Parity vortices *cannot* exist.